

School Mathematics Mastery Test and Preservice Mathematics Teachers' Mathematics Content Knowledge

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Abstract: As part of the effort to equip preservice mathematics teachers with the necessary secondary school mathematics content that they would need to deliver mathematics lessons, a School Mathematics Mastery Test was implemented for all preservice mathematics teachers in the Postgraduate Diploma in Education programme in the National Institute of Education. This paper describes the rationale and the different phases in the implementation of this test, sample test questions, and the performance of the preservice teachers in the different batches since its implementation. The implication of the implementation of this mathematics content test and the future direction of study on preservice teachers' mathematics content knowledge are also discussed.

Key words: Preservice teacher education; Mathematics content knowledge; School mathematics mastery test

Introduction

Preservice teachers from the Postgraduate Diploma of Education programme with specialization to teach in secondary schools (hereafter abbreviated as PGDE (Sec)) and with mathematics as one of their curriculum subjects are expected to become proficient at teaching mathematics at the secondary school level after they have graduated from the National Institute of Education (NIE), the sole teacher education institute in Singapore. But what does it take for teachers to teach mathematics proficiently?

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There are different views amongst mathematics educators on what teachers need to know about teaching mathematics proficiently (Cooney, 1999; Elbaz, 1983; Graeber, 1999; Kilpatrick, 2001; Shulman, 1986). Despite this, most agree that teachers need to have two important kinds of knowledge: content knowledge and pedagogical content knowledge. According to Shulman, content knowledge includes the knowledge of concepts and procedures covered in the subject, in this case mathematics, and the relationships amongst them. Pedagogical content knowledge is about “the ways of representing and formulating the subject that make it comprehensible to others” and “[the] understanding of what makes the learning of topics easy or difficult” (Shulman, 1986, p.9).

Traditionally, in a short teacher education course such as the PGDE (Sec) programme, which lasts for twelve months, the assumption is that preservice teachers have already acquired some level of competency in content knowledge during their tertiary education before coming into NIE. In the curriculum studies (CS) mathematics module, which is compulsory for preservice teachers training to teach mathematics, the emphasis is placed primarily on pedagogy training. Here, preservice teachers will build on their mathematics content knowledge to develop their pedagogical content knowledge. There are thus no mathematics content courses taught in the PGDE (Sec) programme.

In recent years where the admission criteria has been broadened to widen the net for recruitment, the level of content knowledge in the cohorts of preservice mathematics teachers in the PGDE (Sec) programme has appeared to decline. Feedback from tutors teaching the CS mathematics module has revealed that they no longer just focus on pedagogical content knowledge; the tutors also need to deal with the preservice teachers’ mathematics content knowledge too. In addition, anecdotal evidence from Senior Teachers and Heads of mathematics departments from the various secondary schools have also pointed to preservice teachers’ poor teaching performance due to a lack of content mastery. Such a situation was unheard of in the past because preservice mathematics teachers in the PGDE (Sec) programme would have come from the Science or Arts faculty, and would have read a substantial number of mathematics modules at the university.

The recent profile of preservice mathematics teachers has shown much variation in their educational background. A large number of those assigned to the CS mathematics module are from the Engineering or Business faculty. For example, in the July 2001 cohort of 374 preservice mathematics teachers, 31% were engineering graduates and 13% were business graduates; only 28% were mathematics majors. Furthermore, recent intakes saw a substantial number of career change preservice teachers who joined teaching after some years in the workforce. Moreover, with the

broadening of university curriculum at the local universities, even those who had taken some modules in mathematics may not have read the relevant modules necessary for a solid understanding of the content needed at the secondary school levels.

In addition, since 1996, the PGDE (Sec) programme has been admitting preservice teachers whose highest mathematics qualification is at least Grade B in A-level mathematics to teach Lower Secondary mathematics as a second curriculum subject. The mathematics CS modules were further subdivided under two categories: those preservice teachers who had had sufficient exposure to mathematics in their undergraduate studies were classified under the PGDE (All Sec) mathematics CS classes. This group of preservice teachers is trained to be able to teach the entire secondary school mathematics curriculum. The other preservice teachers who did not take mathematics modules at the undergraduate level were assigned to PGDE (Lower Sec) mathematics CS classes. They are trained to teach lower secondary school mathematics curriculum.

Since the 1990s, NIE has seen a tremendous increase in the number of PGDE (Sec) preservice teachers. In particular, the number of PGDE (Sec) preservice teachers has exceeded 500 in the past few intakes (January and July inclusive). In 2005, for example, there were altogether 642 PGDE (Sec) preservice teachers (January and July intakes inclusive) with 496 (77%) doing PGDE (All Sec) mathematics and 146 (23%) doing PGDE (Lower Sec) mathematics.

In the light of the deficiencies in preservice teachers' mathematics content knowledge, this paper describes the action taken by the Mathematics and Mathematics Education Academic Group in NIE to enable the preservice teachers to revise their mathematics content through the imposition of the School Mathematics Mastery Test (SMMT), which will be described in greater detail in the next section. It should be noted that we do not establish the claim that the SMMT is a tool to improve the teachers' mathematical content knowledge; it motivates the preservice teachers to revise and recall the secondary school mathematics concepts. This paper also provides the statistics on the preservice teachers' performance in the SMMT since its implementation.

Development of the School Mathematics Mastery Test (SMMT)

In July 2001, as a first step in addressing the lack of content mastery problem, the PGDE (Sec) team conducted a Mathematics Competency Test for all the PGDE (Sec) preservice teachers taking curriculum studies mathematics (hereafter abbreviated as CS mathematics). The test was based mainly on items from O-level

syllabus mathematics papers. About a quarter of the preservice teachers had obtained less than half the maximum score. In particular, the groups with AO-level mathematics or polytechnic mathematics as their highest mathematics attainment at school level performed extremely poorly. The test results confirmed our initial observations that the content knowledge of some of the PGDE (Sec) preservice teachers falls below the standards we expect for secondary mathematics teachers.

The idea of having PGDE (Sec) CS mathematics preservice teachers sit for a Mathematics Proficiency Test (MPT) was mooted and four models were considered:

1. Use MPT as a qualifying test to disqualify and reject candidates who fail.
2. Use MPT to identify preservice teachers who need to take a separate module on mathematics content during their training at NIE.
3. Use MPT to inform preservice teachers and school administrators on the preservice teachers' proficiency level of secondary school mathematics. The grade for the test can be separately reflected in an 'appendix' to the preservice teacher's academic transcript.
4. Use the passing of MPT as a pre-requisite to passing the PGDE (Sec) CS mathematics course.

There were reservations for the first three models. For model 1, having a qualifying test may turn away those who want to teach mathematics. The reduced intake may not meet the high demand from the schools for mathematics teachers at that time. For model 2, running a separate module for preservice teachers would mean a strain on the existing mathematics staff. Moreover, the content level is below what is expected at a tertiary institution. For model 3, a grade on an 'appendix' to the preservice teacher's transcript was deemed as not sufficient motivation for preservice teachers to want to pass the MPT well. In addition, if a preservice teacher passes the CS mathematics module but has a poor grade for the MPT, it may send an ambiguous signal with respect to his/her qualification to teach secondary mathematics. Model 4 was finally adopted as most colleagues agreed that basic proficiency for secondary school mathematics is essential for teachers and failure of the MPT should rightfully render the preservice teacher not qualified to teach secondary mathematics.

The proposal for MPT was accepted and implemented in July 2003 as the assessment of a zero academic unit module entitled Essential Mathematics for Secondary School Teachers. The MPT was also renamed as School Mathematics Mastery Test (SMMT).

Teachers' Mathematics Content Knowledge and the Objective of SMMT

Mathematics content knowledge is widely regarded as a critical attribute of mathematics teachers (Chapman, 2005). As seen in the case of the US curriculum, the quality and the rigor of the mathematics curriculum is correlated to the subject knowledge of the teachers (Schmidt, 2002). However, many teachers are not adequately prepared in the subject matter they will be teaching (Noddings, 1998). Studies have also shown examples of teachers who wanted to do a good job in teaching mathematics but faced many problems that were largely due to their mathematics preparation (see, for example, Hutchinson, 1997). It can be accepted that having strong content knowledge is a necessary, though not sufficient, condition for good teaching. Good teaching involves much more than a teacher being competent in mathematics. In fact, Usiskin (2001) asserts that "to teach well, a teacher of mathematics should know a great deal of mathematics. The higher the level taught, the more the teacher needs to know" (p. 86). From his perspective, the "great deal of mathematics" does not merely refer to the mathematics content knowledge the teachers acquired during their undergraduate mathematics modules; it refers to the entire branch of mathematics that forms what he calls teachers' mathematics. This includes explanation of new ideas, alternative ways of approaching problems, including ways with and without calculator and computer technology and how ideas studied in school relate to ideas students may encounter in later mathematics study.

On the other hand, researchers have also challenged the belief that the more a teacher knows about his subject, the more effective he can be. For instance, Begle (1979) believes that "the effects of a teacher's subject matter knowledge and attitudes on student learning seem to be far less powerful than many of us assumed" (p. 51). If a teacher's subject matter knowledge is measured by the number of modules of undergraduate mathematics, then according to Ball (1991), research has shown weak correlation between teachers' subject knowledge matter and students' performance in mathematics.

In view of the relevance of the content and the belief that it is not as important to 'upgrade' preservice teachers' content to the undergraduate mathematics knowledge as the "teachers' mathematics" to include "explanation of new ideas, alternative ways of approaching problems..." (Usiskin, 2001), the mathematics content to be tested for the SMMT only includes O-level Mathematics and Additional Mathematics knowledge, albeit from a higher perspective. The preservice teachers must already have acquired mathematical knowledge of the O-level Mathematics when they were students. According to Jaworski and Gellert (2003), this extensive

knowledge is largely limited because it is based mainly on their experience as students. In this sense, the SMMT can serve as a “reflection” component (Jaworski & Gellert, 2003) for the preservice teachers on the secondary school mathematics content.

The goal of the SMMT is to provide a mechanism that will motivate preservice teachers to revise, self-study and build up their secondary school mathematics content up to a mastery level adequate to meet the demands in teaching mathematics in the secondary schools. It is expected that, for most preservice teachers, the process will take time and any short intensive module will unlikely be pedagogically effective in helping them attain the desirable proficiency level or (more ambitiously) make the leap to ‘university-level’ mathematics.

As the SMMT only includes O-level mathematics content, the test should not be used as a qualifying test for course admission, programme placement or any other similar purposes. In addition, the results of SMMT should not be used to predict the performance of the preservice teachers in the PGDE (Sec) CS module or during practicum, since teaching performance depends on many factors and not solely on content knowledge. A good preservice mathematics teacher must be able to pass the SMMT although a candidate who passes the SMMT might not necessarily be a good classroom mathematics teacher. On the other hand, a preservice mathematics teacher who is not able to clear the SMMT after all the attempts allowed in spite of all the forms of assistance provided demonstrates a lack of content knowledge and is not likely to be able to cope with secondary school mathematics teaching.

Organization and Structure of SMMT

An SMMT committee was formed comprising three staff members from the PGDE (Sec) team. The committee chair serves as the coordinator in charge of liaison work with Ministry of Education (MOE) and the Foundation Programme Office (FPO) of NIE as well as coordination for setting the papers, conducting the ‘content upgrading’ module, organizing the logistics of the tests (deployment of part-time staff for invigilation, marking and mark entry) and checking the marked test scripts. The other members of the committee assist in checking the SMMT papers, invigilation and other related administration matters. Sometimes, help from tutors of the PGDE (Sec) team is enlisted in the setting of questions and invigilation.

A “pass” in the SMMT, being a mastery test of the O-level mathematics content knowledge, would mean that a candidate has reached a “mastery level”, which was decided by the SMMT committee as having obtained a score that is equivalent to a distinction according to the O-level standard. It was agreed that merely scoring

more than 50% would not indicate that the candidate has sufficient content knowledge for teaching.

The structure of SMMT is very different in the two phases of implementation. It will be seen in the Results and Discussion section that the structure does have an important impact on the performance of the preservice teachers on SMMT.

Phase One (January 2003 to January 2005)

A pilot SMMT was conducted with the January 2003 cohort. For this experimental batch, there was no consequence for not clearing SMMT. For the subsequent cohorts, which began in July 2003 onwards up to January 2005, SMMT was implemented as a content upgrading module to be passed within three attempts as part of the course requirement for CS mathematics. It is also a prerequisite for awarding the postgraduate diploma in education. This is essentially model 4 discussed under the Development of the School Mathematics Mastery Test (SMMT) section.

After the first attempt at SMMT (henceforth, Attempt One), preservice teachers who passed the test would be exempted from the subsequent tests and deemed to have passed the module. Preservice teachers who did not manage to clear the first attempt were required to sit the test a second time (hence, Attempt Two), which was scheduled either at the end of the First Semester or at the beginning of the Second Semester. For preservice teachers who failed to clear the SMMT by Attempt Two, online help in the form of self-paced learning was provided for the preservice teachers. During the period when the online help was provided, the SMMT coordinator also conducted a series of three lectures clarifying the errors and misconceptions common to secondary school mathematics. Eventually, all preservice teachers were expected to pass the SMMT by Attempt Three.

Phase Two (July 2005 to present)

In July 2005, after an NIE curriculum review conducted by FPO to streamline the different content upgrading modules across subjects, it was decided that content upgrading be moved before the PGDE Programme proper. Effectively, this meant de-linking the SMMT and the PGDE (Sec) mathematics course and that the performance in SMMT would no longer have any bearing on the awarding of the postgraduate diploma.

For the July 2005 cohort, four attempts were provided. The first attempt, which served as an exemption test, occurred about two months before the start of the PGDE course. All those who failed or missed the first attempt had to attend a three-day intensive content upgrading module before sitting the second attempt. For those who failed in the second attempt, two more attempts were provided while they were doing their PGDE course. The name list of the preservice teachers who failed after four attempts was submitted to the Ministry of Education for appropriate follow-up actions such as compulsory in-service content upgrading.

There were various administrative problems associated with the implementation of the content upgrading modules before the commencement of the PGDE course. MOE and FPO finally agreed to move the content upgrading modules back to term time starting from January 2006. The PGDE (Sec) structure was modified to include a six-week period for content upgrading before the commencement of the CS modules but the content upgrading module is still de-linked and considered separate from the PGDE programme.

From the July 2005 episode, the PGDE (Sec) mathematics team learnt that our preservice teachers who were weaker in mathematics content could not be helped through content upgrading modules conducted in a very intensive manner. Rather, many of them picked up the content through self-study and also through sufficient exposure to the course materials presented in the CS mathematics module. In other words, the time for self-study and revision is necessary and conceptual understanding of the mathematics content may be reinforced through the CS mathematics tutorial workshops. The six-week period for content upgrading is too short to conduct two rounds of SMMT, one to identify the target preservice teachers for content upgrading and another at the end of the content upgrading sessions for assessment of the module. Hence, for the January 06 cohort, the SMMT committee decided to conduct only two SMMT attempts, one at the end of the first CS mathematics module, and another at the end of the second module. After reviewing the results of the January 06 cohort (see the Results and Discussion section), an earlier round of exemption test was reinserted from July 06 onwards not only to notify preservice teachers who lacked content mastery to work on their content early, but also to help tutors identify preservice teachers who need more attention and assistance.

Format and Sample Questions of the SMMT

The SMMT is a two-hour paper consisting of about 18 to 20 questions of varying length. For each round of testing, two sets of SMMT papers are generated: one set

for those doing PGDE (All Sec) mathematics and another set for those doing PGDE (Lower Sec) mathematics.

For PGDE (All Sec) preservice teachers, the test content covers all the topics from both Mathematics and Additional Mathematics in the current mathematics syllabus (see <http://seab.gov.sg> for the detailed mathematics syllabus). For PGDE (Lower Sec) preservice teachers, the test content covers the entire mathematics syllabus. Even though these preservice teachers are trained to teach at lower secondary level only, they are also expected to be familiar with the upper secondary school mathematics content in order to be able to plan their lessons more effectively from a more comprehensive perspective.

A table of specifications is drawn up in setting each set of SMMT paper. Approximately 50% of the questions deal with computation while the remaining 50% of the questions test the preservice teachers on mathematical concepts and non-routine problem solving. Candidates are expected to demonstrate mastery level in order to clear the paper. The table of specifications for the different SMMT papers is maintained so that the level of difficulty of the SMMT paper does not vary much across different cohorts of preservice teachers.

In the remaining section, samples of test questions in each strand of mathematics are provided to give the reader an idea of the content being tested. A total of six questions from the previous SMMT papers are presented for discussion here.

Sample Questions

In this section we present some sample SMMT questions, which show the various concepts that the preservice teachers are expected to handle, as well as the questions' level of difficulty in comparison with the O-Level examination questions. These sample questions are made available to the preservice teachers through the NIE intranet. All the cohorts of the preservice teachers are advised to attempt the questions before sitting for SMMT.

Arithmetic

Q1. Consider the nine numbers

$$0.4, \frac{3}{2}, \sqrt{2}, \pi, 5, 9, 17, 40, 121.$$

Write down (a) the two prime numbers; (b) the two square numbers; (c) all the irrational numbers; (d) all the complex numbers

- Q2. Three bus services operate from the same depot. The first service leaves at 10-minute intervals, the second at 15-minute intervals and the third at 25-minute intervals. All three services leave the depot together at 08 00. Find the time when the three services next leave the depot together.

Commentary. As can be seen from Q1 above, candidates are expected to be familiar with the entire number system; even though complex numbers are not in O-level curriculum, preservice teachers are still expected to be familiar with the entire number system and their relationship. Candidates are also required to be able to solve word problems related to number concepts such as lowest common multiple, as illustrated in Q2 above.

Algebra

- Q3. Find all value(s) of a for which the equation $(x - a)^2 + (x - 3)^2 = 0$ has real solution(s) for x .

Commentary. Besides being able to solve different types of algebraic equations required in the O-level curriculum, preservice teachers are required to analyse any algebraic equation given to them as illustrated in Q3 above. If one has a sound understanding of the solution of algebraic equations and quadratic functions, it will be apparent by inspection that the only possible choice of a for which the equation has a solution is when $a = 3$ only. Alternatively, preservice teachers might use the discriminant of the quadratic equation to conclude that $a = 3$ is the only possible choice. However, this alternative solution is much more tedious. It is important for preservice teachers to know how to generate different possible solutions, and in particular, to arrive at the shortest possible solution.

Geometry

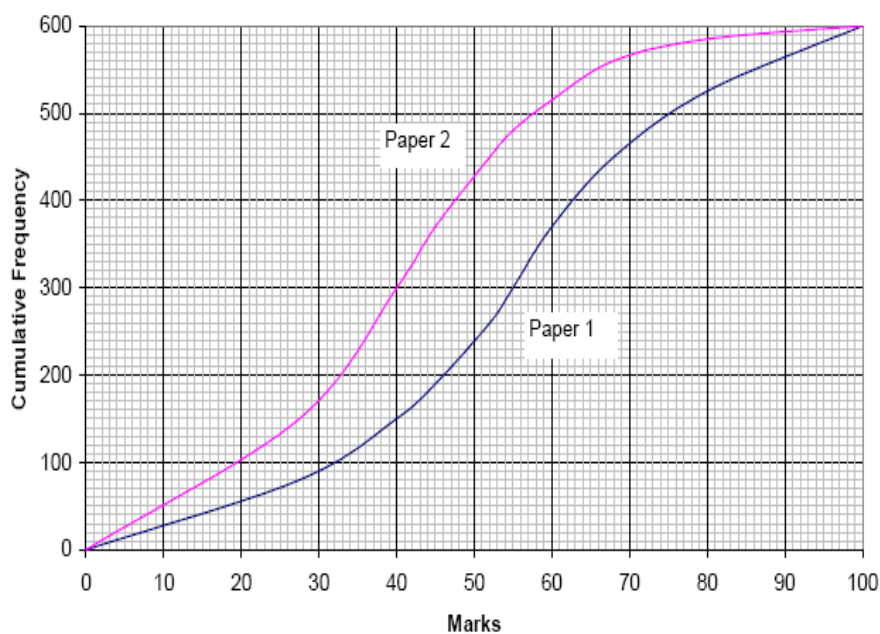
- Q4. (a) Let ABC be a triangle with M and N as the midpoints of the sides AB and AC respectively. Show that MN is parallel to BC and has half the length of BC.
- (b) Let ABCD be a quadrilateral. Let M, N, P and Q be the midpoints of AB, BC, CD and DA respectively. By using (a) above, prove that MNPQ is a parallelogram.

Commentary. Geometry proofs are required in the new Additional Mathematics curriculum (see, Ministry of Education, 2007). As such, preservice teachers are

required to be able to write down simple mathematical proofs that are required in the O-level mathematics syllabuses.

Probability and Statistics

- Q5. Six hundred candidates took a mathematics examination which consisted of two papers. Each paper was marked out of 100. The diagram shows, on the same axes, the cumulative frequency curves for Paper 1 and Paper 2.



- Use the graph for Paper 1 to estimate
 - the median,
 - the interquartile range,
 - the number of candidates who scored more than 45 marks.
- A candidate scored 60 on Paper 1. Use the two graphs to estimate this candidate's mark on Paper 2.
- State, with a reason, which you think was the more difficult paper.

Commentary. As this question on statistics makes clear, the focus of SMMT is not so much a test of the preservice teachers' skills in graph construction. Rather, the emphasis is on the interpretation of data and application of statistical concepts in real-life situations.

Calculus

- Q6. A right circular cylinder open at one end and closed at the other end is to be constructed with surface area 100 cm^2 . What is the largest possible volume of the cylinder?

Commentary. In calculus, preservice teachers are expected to be able to apply their knowledge of calculus such as applying differentiation to find the maximum and minimum values in the example of Q6 to solve an entire non-structured problem, rather than merely solving a structured question with several parts as evidenced in many past-year O-level examination papers. This ability is important to preservice teachers for teaching and construction of test items.

Online and Other Support for Preservice Teachers

Generally, many preservice teachers who fail to clear the SMMT Attempt One can pass in Attempt Two by revising their content knowledge more thoroughly. Moreover, between Attempt One and Attempt Two, preservice teachers also have the opportunity of checking concepts and clearing doubts during their CS mathematics tutorial workshops. Thus, only preservice teachers who fail the SMMT in the penultimate attempt are given additional help to boost their mathematics content.

Between the end of SMMT penultimate attempt and the start of the last attempt of SMMT Attempt, online help is provided for the preservice teachers. The online help consists of links to various websites on the independent learning of the various secondary school mathematics topics. In the online help, materials such as sample SMMT questions (as discussed in the Sample Questions subsection) with proposed answers and notes on selected Additional Mathematics topics are provided for the preservice teachers.

Preservice teachers are expected to revise their O-level content for conceptual understanding rather than use drill-and-practice to prepare for SMMT in the way that they may have prepared for national examinations using past year papers.

There are many resources available for O-level mathematics in the form of textbooks and references. Additional resources (Toh, 2005; 2006) have also been developed by the SMMT coordinator. The long-term goal of the SMMT committee is to build up a data bank for future use when a sufficiently large number of questions have been collated.

Preservice Teachers' Performance in SMMT

This section presents the statistical figures on the preservice teachers' performance over the several years of SMMT since its implementation. The figures collected over the past years aim to answer the following questions:

1. What is the overall passing rate of the preservice teachers who can clear the SMMT with and without the help provided by NIE?
2. What is the overall percentage of preservice teachers who have difficulty clearing the SMMT even with the provision of help by NIE?

The number of attempts in which preservice teachers are allowed to take the test has not been constant since the implementation of SMMT. As such, it is important to compare across different batches by identifying the corresponding attempts. Specifically, we shall classify the preservice teachers in the various attempts under three broad categories:

- A: preservice teachers who passed the SMMT without any official help from NIE;
- B: preservice teachers who passed the SMMT with official help provided by NIE; and
- C: preservice teachers who did not pass the SMMT after exhausting all the possible attempts.

Here, official help provided by NIE includes help in the form of online resources, additional reading materials, lectures on error analysis and misconceptions in secondary school mathematics as well as in-class and out-of-class assistance of tutors from the PGDE (Sec) team.

Results and Discussion

Results for the PGDE (All Sec) Preservice Teachers

From the results of Table 1 below, it can be seen that in all the batches, with the exception of the July 2005 cohort, at least 60% of the preservice teachers have been

classified under Cat A. That is, they were able to clear the SMMT through self-revision, without the official help provided by NIE.

Table 1
Performance of Preservice Teachers in SMMT for PGDE (All Sec) Paper

Batch	No. of Preservice Teachers	Cat		Cat		Cat	
		A	Percent	B	Percent	C	Percent
Jan 2003 [#]	129	83	64	43	33	3	2
July 2003	368	293	80	75	20	0	0
Jan 2004	145	91	63	54	37	0	0
July 2004	321	201	63	120	37	0	0
Jan 2005	183	126	69	57	31	0	0
July 2005*	313	122	39	178	57	13	4
Jan 2006 *	126	85	67	29	23	12	10

[#] Pilot batch

* SMMT de-linked from PGDE (Sec) CS mathematics

The exceptionally low percentage of preservice teachers under Cat A for the July 2005 batch could be attributed to the changes in the administrative procedure for that cohort as discussed in the Organization and Structure of SMMT section. The July 2005 batch was the first batch for which the SMMT was conducted before the commencement of the PGDE term. Many preservice teachers were not informed of the first attempt and missed the test. Together with those who failed in the first attempt, they were required to attend intensive content revision lectures shortly after the first attempt. Consequently, a large number of preservice teachers were classified under Cat B even though they might have the potential to clear the SMMT without much official help. Thus, the data for the July 2005 batch might not be reflective of the general ability of the preservice teachers who sat the PGDE (All Sec) paper.

From the same table, it can be seen that from July 03 to January 05, no preservice teacher was classified under Cat C. The result shows that the preservice teachers, either on their own or with the external help provided by NIE, were able to clear the SMMT before their graduation from the institute.

For July 2005 and January 2006 batches, 4% and 10% respectively of the preservice teachers failed to clear the SMMT ultimately. This increase in the percentage of Cat C may be attributed to the change in the structure of SMMT. There are two plausible explanations: (1) Prior to July 2005, the SMMT was a compulsory module that preservice teachers must pass before their graduation from the institute as a fully qualified teacher. After July 2005, the SMMT was de-linked from the PGDE CS mathematics course. Thus, there is a lack of motivation for some preservice teachers to clear the SMMT amongst all other compulsory modules; (2) In January 2006, the number of attempts was reduced to two, instead of three to four attempts that were possible in the past. Thus the preservice teachers could have insufficient experience preparing for the SMMT.

Results for the PGDE (Lower Sec) Preservice Teachers

From the results of Table 2, with the exception of the January 2004 and July 2005 cohorts, at least 60% of the preservice teachers have been classified under Cat A. That is, they were able to clear the SMMT without the official help provided by NIE. The reason for the exceptional performance of the July 2005 batch was similar to the reason for the PGDE (All Sec) performance mentioned in the preceding subsection.

Table 2
Performance of Preservice Teachers in SMMT for PGDE (Lower Sec) Paper

Batch	No. of Preservice Teachers	Cat		Cat		Cat	
		A	Percent	B	Percent	C	Percent
Jan 2003 [#]	35	24	69	10	29	1	3
July 2003	98	86	88	12	12	0	0
Jan 2004	19	3	16	16	84	0	0
July 2004	112	81	72	31	28	0	0
Jan 2005	24	16	67	8	33	0	0
July 2005*	122	28	23	94	77	0	0
Jan 2006*	36	23	64	4	11	9	25

[#] Pilot batch

* SMMT de-linked from PGDE (Sec) CS mathematics

The exceptional performance for the January 2004 PGDE (Lower Sec) cohort could be simply due to statistical variation as the size of the cohort was small. Although a few preservice teachers from January 2004 cohort complained that the paper was difficult, the percentage of higher order thinking questions and the distribution of questions over the different topics tested did not differ from those in the other years.

As in the case for the PGDE (All Sec) preservice teachers, with the exception of the January 2006 batch, very few preservice teachers were classified under Cat C. The reason for the exceptional performance for the January 2006 cohort is similar to that for the January 2006 batch for the PGDE (All Sec) preservice teachers.

Conclusion

Since its implementation in July 2003, the SMMT has served three main functions. First, it serves to send a clear signal to our preservice teachers about the importance of content mastery in mathematics teaching. Second, it re-assures school educators and parents that realistic measures have been taken to tackle the problem of declining mathematics proficiency level amongst the preservice teachers. Finally, it provides a mechanism to motivate preservice teachers weak in mathematics to attain content mastery during their preservice course.

The results of the SMMT for the previous cohorts have shown that within the widening criteria of recruiting teachers to teach mathematics in the secondary school, there is a fairly high percentage of the preservice teachers who are still able to clear the SMMT on their own. Another significant portion of preservice teachers are able to clear the SMMT with official help provided by NIE. Only a very small proportion of preservice teachers are not able to clear the SMMT even with the help given.

The SMMT may be criticized for its coverage of basic content (practically the O-level mathematical content knowledge). However, based on the data we have collected, official help is still needed by a significant proportion of the preservice teachers before they can clear such a test involving secondary school mathematics content knowledge.

While the SMMT is a mechanism for preservice teachers to work on their secondary school mathematics content, very few studies on the testing of preservice teachers' mathematics content knowledge were previously available. Currently, there are

some studies which are related to the testing of preservice mathematics teachers' mathematical content knowledge, as a part of the larger scale comparative study of teacher education with a focus on the preparation of teachers of mathematics at the primary and lower secondary levels. Two such studies which some academic staff members from NIE are involved are: (1) Teacher Education and Development Study – Mathematics, which is also abbreviated and commonly known as TEDS-M, (see <http://teds.educ.msu.edu/default.asp>), and (2) the International Comparative Study in Mathematics Teacher Training (ICSMTT) conducted by the Centre of Innovation in Mathematics Teaching (CIMT) of the University of Plymouth. Further results on preservice teachers' mathematics content knowledge in relation to the teacher education programmes can be established after those studies.

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